

$$M/m = x^2/(r-x)^2$$

$$\text{Root}(M/m) = k = x/(r-x)$$

$$k(r-x) = x$$

$$(1+k)x = kr$$

$$x = k/(1+k) \times r$$

$$k = \text{Sqrt}(5.5\text{E}25/3.4\text{E}21) = 127.1867547672921$$

$$x = 127.1867547672921/(1 +$$

$$127.1867547672921) = 0.9921988819998$$

$$x = 0.9921988819998 \times (4.3\text{E}7 + 2.5\text{E}6) =$$

$$4.5145049130991\text{E}7$$

from planet centre

$$y = (4.3\text{E}7 + 2.5\text{E}6) - 4.5145049130991\text{E}7 =$$

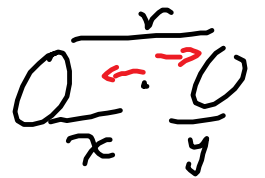
$$354,950.8690090105$$

from moon centre

$$F_{g1} = F_{g2}$$

$$\frac{M_m}{x^2} = \frac{M_e}{y^2}$$

$$\frac{M_1}{x^2} = \frac{M_2}{y^2}$$



$$x + y = r$$

Gravitational Energy Handout

Hand back Quiz

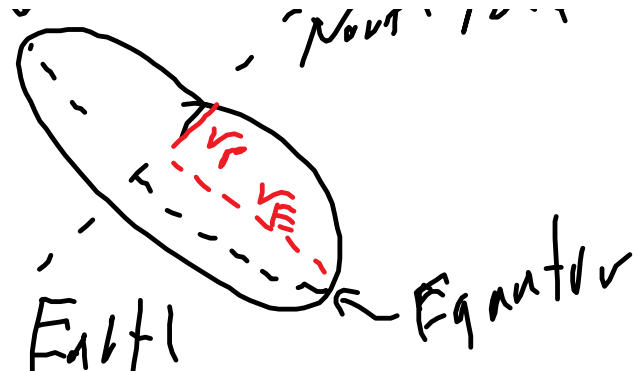
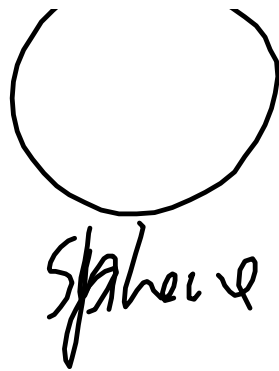
Test next Wednesday

Q 1

North pole is closer to the centre (smaller r)

because the Earth is not a perfect sphere (bulges in the centre)





$$r_p < r_E$$

$$\therefore F_{gp} > F_{gE}$$

but this only accounts for less than half
of the observed effect
9.78 to 9.82 N/kg
What accounts for the rest.

At the equator, all objects are partially in
freefall due to the circular motion (spin).
What is the magnitude of this effect?

$$a = 4\pi^2 r / T^2 = 4\pi^2 (6.38 \times 10^6 \text{ m}) / (24 \text{ h})^2$$

$$a = 4 \times 3.14159^2 \times (6.38 \text{ E}6) / (24 \times 3600)^2 =$$

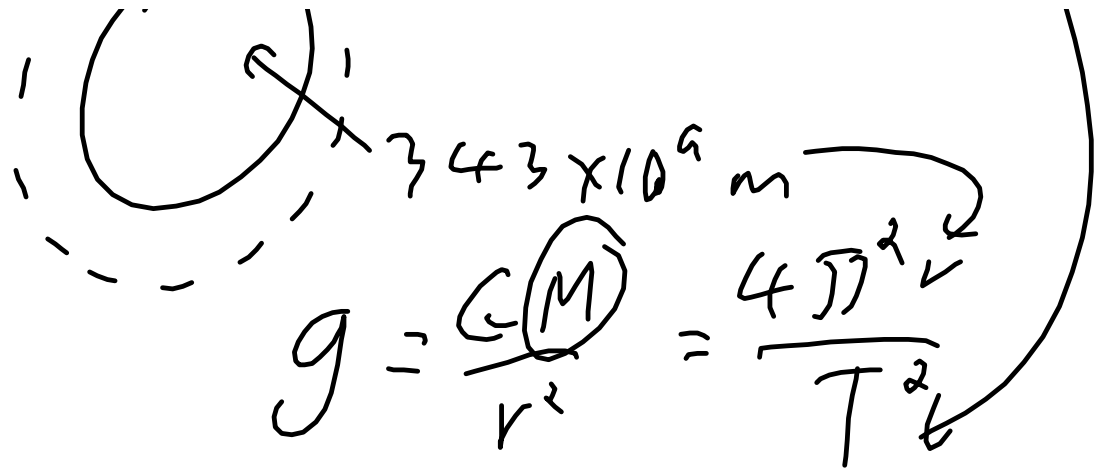
$$0.03374 \text{ m/s}^2 \text{ or N/kg}$$

this is the difference btween $9.82 - 0.03 =$
 9.79

Q2

102 minutes

$$T = \cancel{567 \times 10}$$



$$4\pi^2 r / T^2$$

$$4 \times 3.14159^2 \times (3.43 \text{E}6) / (102 \times 60)^2 = 3.61535 = \underline{3.62 \text{ N/kg}}$$

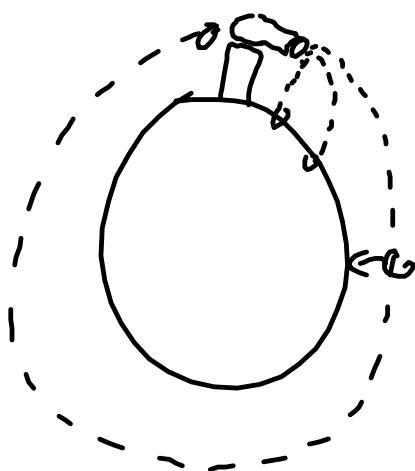
$g=a$ if no other forces

Q3

$$a = 4\pi^2 r / T^2$$

$$4 \times 3.14159^2 \times (3.8 \text{E}8) / (27.3 \times 24 \times 3600)^2 = 0.0027 \text{ m/s}^2$$

Q4



$$F_g = F_c$$

$$mg = m \frac{v^2}{r}$$

$$v = \sqrt{gr}$$

on orbit
 $g \hat{=} 9.8$

$$v = \text{Sqrt}(9.81 \times 6.38 \times 10^6) = 7,911.24516116142$$

$7.9 \times 10^3 \text{ m/s}$ or 7.9 km/s

$$6.67 \times 10^{-11} \times 5.98 \times 10^{24} / (6.38 \times 10^6)^2 = 9.7991$$

Q5 $4\pi^2 r / T^2$

$$4 \times 3.14159^2 \times (1.5 \times 10^{11}) / (365.25 \times 24 \times 3600)^2 = 0.00595 \text{ m/s}^2$$

Q6 moon is accelerating at a smaller rate towards the Earth than the Earth is towards the Sun.

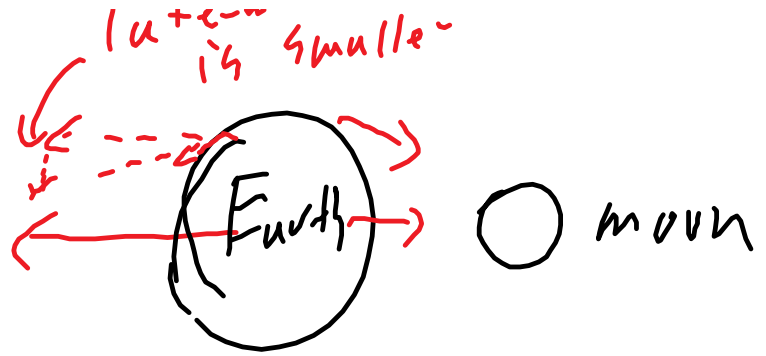
0.0027 m/s^2 vs 0.00595 m/s^2

Hey but the moon influences tides more than the sun, what's the deal?

The Sun is pulling more but the Earth is in freefall towards both Sun and moon, so the tides are caused by the difference in forces at different point. Think of it as the tangential components of the force.



lateral component
is smaller



Eric - How do you find the mass of the Earth?
(before Google?)

use G and r and F_g

How did they determine G ?

Cavendish experiment of massive lead balls,
some hanging on cables some close by and
measure the attraction.

Q7 a , v and T are same

F_c is double $F = ma$

Q8 $E_g = -GMm/r = -$

$(6.67E-11 \times 5.98E24 \times 1) / (6.38E6) = -6.2518E7$

$-6.25 \times 10^7 \text{ J}$

Q9 $\Delta E_g = -GMm (1/r_f - 1/r_i)$ not h !!!!!!!

$-(6.67E-11 \times 5.98E24 \times 1) \times ((1/3.8E8) - (1/6.38E6)) = 6.1468E7$

$6.15 \times 10^7 \text{ J}$ ignoring the pull of the moon

$$-(6.67\text{E-}11 \times 7.35\text{E}22 \times 1) \times (-$$

$$(1/3.8\text{E}8) + (1/6.38\text{E}6)) = -755,507.9067$$

$$6.1468\text{E}7 - 755,507.9067 = 6.0712\text{E}7$$

$6.07 \times 10^7 \text{ J}$ including the pull of the moon

Q10 escape velocity

$$E_t = E_k = -E_g$$

$$\frac{1}{2}mv^2 = GMm/r$$

$$v = \text{Sqrt}(2 \times 6.67\text{E-}11 \times 7.35\text{E}22 / 1.74\text{E}6) =$$

$$2,373.815494093844$$

$$2.4 \text{ km/s}$$

$$\text{Q11 } E = GMm/r =$$

$$6.67\text{E-}11 \times (7.35\text{E}22) \times (1000) / 1.74\text{E}6 = 2.8175\text{E}9$$

$2.8 \times 10^9 \text{ J}$ required to remove 1000kg from moon

Earth

$$6.67\text{E-}11 \times (5.98\text{E}24) \times (1000) / 6.38\text{E}6 =$$

$$6.2518\text{E}10$$

$$6.25 \times 10^{10} \text{ J}$$

$6.2518\text{E}10 / 2.8175\text{E}9 = 22.1892$ time the energy to release materials from Earth than moon.

$$R = 2GM/c^2 \text{ derive from } v_{\text{escape}} = c$$

$$v = \sqrt{2GM/r}$$

$$v^2 = 2GM/r$$

$$r = 2GM/v^2$$

set $v=c$ definition of black hole - gravity is so strong light can't escape at event horizon

$$R = 2GM/c^2$$

$$r = 2 \times 6.67 \times 10^{-11} \times 1.99 \times 10^{30} / (3.00 \times 10^8)^2 = 2,949.6222$$

about 3 km in radius

p147 Q58, 62, 63, 64, 67, 68

p123 Q61, 62 on the test